INTERNATIONAL JOURNAL OF CLIMATOLOGY
Int. J. Climatol. **25**: 1611–1618 (2005)
Published online in Wiley InterScience (www.interscience.wiley.com). DOI: 10.1002/joc.1206

FORECASTING AN INDEX OF THE MADDEN-OSCILLATION

ELIZABETH A. MAHARAJ[a,*] and MATTHEW C. WHEELER[b]
[a] *Department of Econometrics and Business Statistics, Monash University, Australia*
[b] *Bureau of Meteorology Research Centre, Melbourne, Australia*

Received 4 October 2004
Revised 4 April 2005
Accepted 4 April 2005

ABSTRACT

Prediction of a daily bi-variate index of the Madden–Julian oscillation (MJO) is explored using traditional methods of time series analysis. The index is the pair of empirical, orthogonal function (EOF) time series of Wheeler and Hendon, describing the state of the convectively coupled, baroclinic structure of the MJO along the equator. Seasonally varying vector autoregressive (VAR) models of varying order are fitted to the time series and their first differences. The first-order VAR model on the original (nondifferenced) time series was found to be the most satisfactory for forecasting the index beyond a few days. Although this model shows no strong skill advantage over a lagged regression technique, it has the convenience of employing only a single set of equations to make predictions for multiple forecast horizons. Copyright © 2005 Royal Meteorological Society.

KEY WORDS: MJO; bivarite series; VAR models; forecasting skill

1. INTRODUCTION

The benefits of predictions of the Madden–Julian oscillation (MJO) have previously been well described and discussed (von Storch and Xu, 1990; Lo and Hendon, 2000; Waliser *et al*., 2003). For example, the MJO has been linked to intraseasonal variations in tropical cyclone activity (e.g. Hall *et al*., 2001), making MJO forecasts potentially useful for providing warning of their formation up to several weeks in advance.

In a recent paper, Wheeler and Hendon (2004), hereafter WH, developed an index of the MJO intended for real-time monitoring and empirical prediction applications, comprising a pair of time series to describe its state along the equator. Importantly, the use of conventional time filtering to isolate the signal of the MJO was avoided. The properties of the index series were investigated and described, as were their relationship to aspects of synoptic weather, but methods of predicting the series were not provided.

Here we extend the work of WH by exploring the traditional methods of time series analysis to model and predict their daily bi-variate MJO index. In particular, seasonally varying vector autoregressive (VAR) models are fitted to the indices as well as to the series of their first differences, paying special attention to the applicability and skill of each model. The aim is to be able to select one model above all others, and provide a reasonable evaluation of its performance. The primary objective of the selected model will be to produce skilful extended-range forecasts.

A brief summary of WH's MJO index is given in Section 2, and of VAR models in Section 3. Section 4 presents the results, while Section 5 provides conclusions and some discussion with respect to other developed methods of MJO forecasting.

* Correspondence to: Elizabeth A. Maharaj, Department of Econometrics and Business Statistics, Monash University – Caulfield Campus, PO Box 197 Caulfield East Vic. 3145, Australia; e-mail: ann.maharaj@buseco.monash.edu.au

2. THE REAL-TIME MULTIVARIATE MJO (RMM) INDICES

The MJO index comprises the pair of projection time series of the leading empirical orthogonal functions (EOFs) of the combined fields of near-equatorially averaged 850-hPa zonal wind, 200-hPa zonal wind, and satellite-observed outgoing longwave radiation (OLR) data (see WH for details). Noting that OLR is a good proxy for large-scale vertical motion and deep convection, the EOFs describe the vertically oriented circulation cells of the MJO, reminiscent of the original schematic of Madden and Julian (1972); their Figure 16. Projection of daily data (with the annual cycle and components of interannual variability removed) onto these EOFs yields the index time series that vary mostly on the 30- to 80-day timescale of the MJO (e.g. Figure 1). The daily time series thus serves as an effective index of the MJO without the need for conventional time filtering, and are called the *Real-time Multivariate MJO* series 1 (RMM1) and 2 (RMM2). Together, RMM1 and RMM2 provide information about the state of the MJO around the entire tropics, and were shown by WH to be applicable in all seasons. During times of identifiable MJO activity, RMM1 leads RMM2 by about a quarter cycle, indicative of eastward propagation of the MJO along the equator. In this paper, we use RMM1 and RMM2 from the period of 1 January1979 to 10 November 2003, providing 9080 days of observations.

3. VECTOR AUTOREGRESSIVE (VAR) MODELLING

We assume that the bi-variate MJO time series $Y_t = [x_t, y_t]$, where $x_t = \text{RMM1}$ and $y_t = \text{RMM2}$, has been generated from a stationary VAR process of order p

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} + \mu_t \quad (1)$$

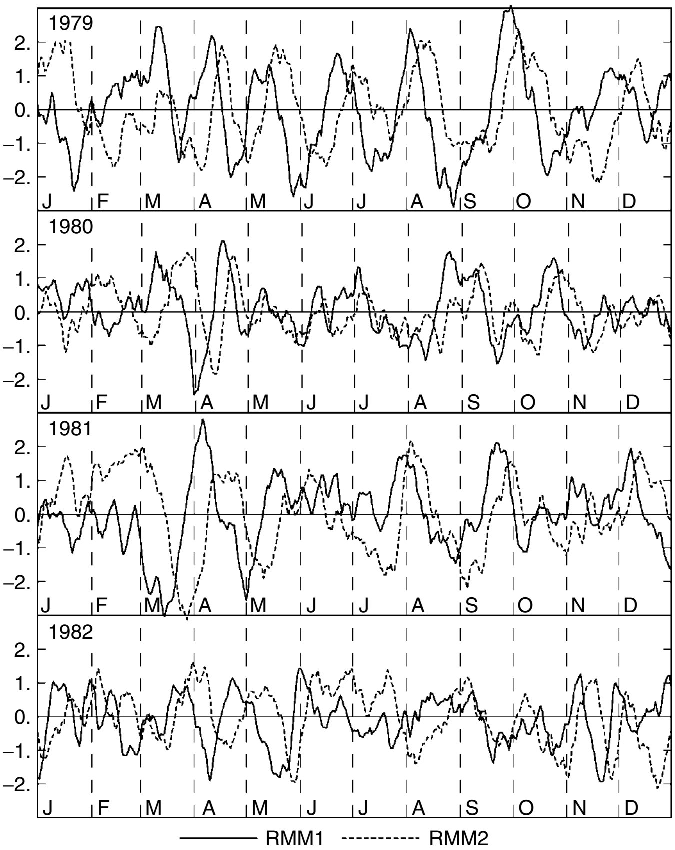

Figure 1. The normalised principal component time series (RMM1 and RMM2) of the leading pair of EOFs as used to represent the MJO, for the period 1 January 1979 to 31 December 1982

where, $Y_{t-1}, Y_{t-2}, \ldots, Y_{t-p}$ are lagged versions of Y_t; $A_1, A_2, \ldots, A_p$ are 2×2 matrices consisting of VAR parameters; and $\mu_t = [\mu_{1t}, \mu_{2t}]$ is a two-dimensional white-noise process.

The VAR(p) process is stationary if the characteristic polynomial

$$det(I_2 - A_1 z - A_2 z^2 - \cdots - A_p z^p) \neq 0 \quad \text{for } |z| \leq 1 \tag{2}$$

i.e. if it has no roots in or on the unit circle. I_2 is the 2×2 identity matrix. If the stationarity condition is not satisfied, the individual series should generally be differenced d times until it is stationary (e.g. Makridakis *et al.*, 1998, p. 347). In practice, differencing of order $d = 1$ or $d = 2$ is usually sufficient. If each of the series has to be differenced d times, then the model for the original bi-variate series is a nonstationary model of the form

$$(1 - B)^d (Y_t - A_1 Y_{t-1} - A_2 Y_{t-2} - \cdots - A_p Y_{t-p}) = \mu_t \tag{3}$$

where B is the backshift operator such that $BY_t = Y_{t-1}$, $B^2 Y_t = Y_{t-2}$, and so on. Given an observed bi-variate series, simple plots of the individual series and their autocorrelation and partial autocorrelation functions can also give an indication whether the time series are stationary, close to being nonstationary, or nonstationary.

The seasonality of the MJO is well documented (e.g. Zhang and Dong, 2004). For example, its globally averaged signal tends to be strongest in the Southern-Hemisphere summer. Thus the possibility of seasonal effects is incorporated into the VAR models by using time-varying parameters (see Lutkepohl, 1991). For this, we use the defined seasons of southern summer (November through April) and northern summer (May through October), as these represent the times of greatest contrast in the behaviour of the MJO (see also Wheeler and Kiladis, 1999). That is, the VAR parameters are defined as

$$[A_{1t}, A_{2t}, \ldots, A_{pt}] = \sum_{i=1}^{2} n_{it} [A_{1i}, A_{2i}, \ldots, A_{pi}] \tag{4}$$

where, the seasonal dummy variables are set as $n_{1t} = 1$, $n_{2t} = 0$ for southern summer, and $n_{1t} = 0$, $n_{2t} = 1$ for northern summer. This is equivalent to fitting independent VAR models for the different seasons.

Given the observed bi-variate MJO series, the appropriate order, p, of the VAR model can be determined by one of many information criteria. Information criteria (see Makridakis *et al.*, 1995, chapter 7) are defined functions of the forecast errors, among other parameters, and are used to select the best fitting model among a small set. For example, we may want to choose from among VAR models of order $p = 1$, 2, 3 or 4. Once each model is fitted to the time series, forecast errors are calculated, and then on the basis of the errors, information criteria are computed and compared among the set of models. In this work, we use the Bayesian information criterion (BIC) derived by Schwarz (1978). The model that returns a minimum value of the BIC is regarded as having the best fit.

While information criteria are useful for determining the fit of a model for the given time period, they are not as useful for assessing the skill the models may have for forecasting the future. To get an indication of this, we validate the models using an independent set, i.e. a set of observations that were not used to develop them. Accuracy measures such as the root mean square (RMS) error are determined for each fitted model in the validation set and the model that returns a better measure of accuracy, say, a lower value of RMS error, is regarded as having the most skill.

Data from 1 January 1979 to 31 December 2000 are used to fit the models, i.e. the estimation set, while the rest are used for validation. One to twenty day-ahead forecasts are computed and the RMS errors and correlations between the actual time series values in the validation set and the forecast values for this set are used to measure the accuracy.

Once the appropriate order model is fitted to the entire observed record, checks are carried out on the residuals of the fit to determine if they follow a white noise process, as assumed above. If, for example, a VAR(1) model is fitted and there is significant residual autocorrelation at lag 2 or 3, then a higher order VAR model might be considered to be more suitable. However, as forecasting is the primary objective in this study, the extent to which the residuals are white noise is deemed to be less important for selecting a model than forecast accuracy.

4. RESULTS

The first four years of the RMM1 and RMM2 series are displayed in Figure 1. While they appear to be mostly stationary about a zero mean, they do display some extended periods in which their multi-month mean drifts away from zero (e.g. from June to November, 1981, RMM1 is more positive). We have thus also explored the first difference time series (not shown). The first differences, while appearing more nearly stationary, are significantly noisier than the original. Obviously, the differencing amplifies the day-to-day noise present in the RMM series.

Examination of the autocorrelation and partial autocorrelation functions of the series and their first differences confirms what we subjectively determined above (Figure 2). That is, (i) the original series are close to being nonstationary because while their first lag partial autocorrelations are close to one, their autocorrelation functions are not dominated by large positive autocorrelations (see, e.g. Makridakis *et al.*, 1995, chapter 7) and (ii) the series of first differences are, to within a good approximation, stationary (the first lag partial autocorrelations are between 0.60 and 0.63 and the autocorrelation function appears to decay exponentially). Hence we have fitted VAR models to both the original series and the series of first differences for validation and comparison.

VAR models of up to order 4 were fitted to the original series and the BIC determined that the VAR(2) model to be a better fitting model than the VAR(1), but not much different from the VAR(3) and VAR(4) models. Since significant, early order partial autocorrelations also give an indication of the appropriate order model to be fitted, the fact that the partial autocorrelations at the first two lags are significant, i.e. outside the 95% confidence limits, verifies that the VAR(2) model may be the most appropriate. However, the strict stationarity condition (Equation (2)) for the VAR(2) and higher order models was not satisfied, so it was decided that further analysis would be carried out for VAR(1) models only (which passed the strict stationarity condition).

The applicability of the time-varying model (Equation (4)) was determined using the hypothesis test as specified by Lutkepohl (1992). This test was passed at all nominal levels of significance, i.e. 1%, 5% and 10%. The VAR(1) model fitted using the original (nondifferenced) series from the test set is:

Southern summer

$$\hat{x}_{t+1} = 0.9561x_t - 0.1207y_t + \mu_t$$
$$\hat{y}_{t+1} = 0.1256x_t + 0.9837y_t + \mu_t$$

Northern summer

$$\hat{x}_{t+1} = 0.9786x_t - 0.1049y_t + \mu_t$$
$$\hat{y}_{t+1} = 0.0936x_t + 0.9545y_t + \mu_t$$

where x_t and y_t represent the observations at time t of RMM1 and RMM2 respectively, while $\hat{x}_{t+1}$ and $\hat{y}_{t+1}$ are the one-day-ahead forecasts for these series. To forecast two or more days ahead, the previous day forecasts, that is, $\hat{x}_{t+h-1}$ and $\hat{y}_{t+h-1}$, where h is the number of days ahead (forecast horizon), are substituted into these equations. The forecast for the error term (μ_t) is 0.

For this time-varying VAR(1) model, the RMS errors of the forecasts from the validation set and the correlation coefficients between the actual observations in the validation set and the forecasts, are given in Table I. It can be seen that the model performs quite well with statistically significant skill, as determined by the correlations, up to day 20. These correlations are significantly different from zero at all nominal levels, i.e. 1%, 5% and 10%, assuming $n - 2$ degrees of freedom where $n = 1410$ (the size of the validation set).

On fitting a VAR(1) model to the series of first differences (this was appropriate since only the partial autocorrelation for the first lag was outside the 95% confidence limits) and obtaining the corresponding

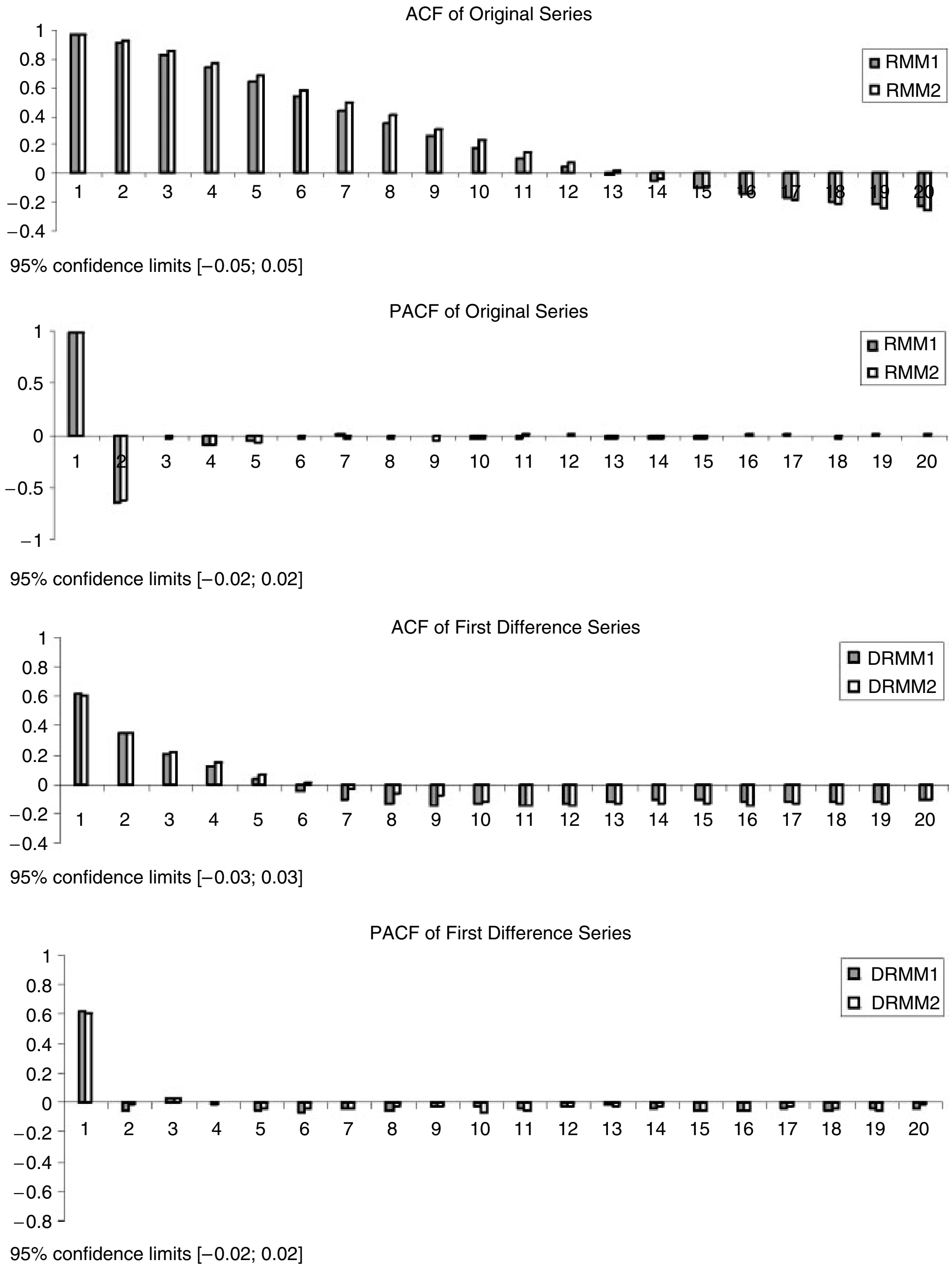

Figure 2. The autocorrelations and partial autocorrelations of RMM1, RMM2 and their first differences, DRMM1, DRMM2. The 95% confidence limits are $\pm 1.96/\sqrt{T}$, where T is the length of the time series under consideration

Table I. Forecast skill statistics (RMS error and correlation) as computed for the validation set of observations

	Original series					First difference of original series			
	RMS		Correlation			RMS		Correlation	
Day	RMM1	RMM2	RMM1	RMM2	Day	RMM1	RMM2	RMM1	RMM2
1	0.2103	0.1880	0.9765	0.9847	1	0.1899	0.1822	0.9816	0.9859
2	0.3571	0.3136	0.9310	0.9566	2	0.3598	0.3332	0.9348	0.9532
3	0.4678	0.4126	0.8799	0.9235	3	0.5120	0.4723	0.8691	0.9063
4	0.5541	0.4933	0.8294	0.8886	4	0.6488	0.6005	0.7904	0.8486
5	0.6227	0.5621	0.7822	0.8525	5	0.7755	0.7176	0.7007	0.7837
6	0.6758	0.6229	0.7411	0.8152	6	0.8912	0.8245	0.6044	0.7141
7	0.7151	0.6797	0.7077	0.7750	7	0.9950	0.9277	0.5063	0.6380
8	0.7431	0.7323	0.6817	0.7327	8	1.0869	1.0277	0.4102	0.5554
9	0.7621	0.7792	0.6621	0.6900	9	1.1622	1.1226	0.3252	0.4696
10	0.7769	0.8192	0.6454	0.6495	10	1.2240	1.2106	0.2511	0.3830
11	0.7901	0.8520	0.6289	0.6129	11	1.2796	1.2890	0.1815	0.3002
12	0.8097	0.8800	0.6064	0.5803	12	1.3297	1.3576	0.1158	0.2235
13	0.8216	0.9046	0.5892	0.5485	13	1.3735	1.4197	0.0560	0.1511
14	0.8337	0.9267	0.5707	0.5179	14	1.4131	1.4769	0.0001	0.0810
15	0.8452	0.9461	0.5518	0.4897	15	1.4504	1.5322	−0.0545	0.0114
16	0.8547	0.9623	0.5339	0.4650	16	1.4867	1.5831	−0.1087	−0.0552
17	0.8632	0.9760	0.5165	0.4433	17	1.5198	1.6288	−0.1591	−0.1171
18	0.8716	0.9876	0.4981	0.4244	18	1.5509	1.6697	−0.2065	−0.1742
19	0.8807	0.9971	0.4783	0.4086	19	1.5791	1.7062	−0.2505	−0.2271
20	0.8906	1.0043	0.4563	0.3960	20	1.6021	1.7378	−0.2874	−0.2743

nonstationary model for the original series, we found much poorer results, as also presented in Table I. With the exception of lags 13 to 16, the correlations are all significantly different from zero at all nominal levels. However, while the skill was comparable to the VAR(1) model on the original series for forecasts out to 2 days, the skill for longer leads was comparatively less, and is worse than guessing (a negative correlation) beyond day 14. Higher order VAR models fitted to the series of first differences did not provide any improvement to the validation set statistics. An explanation for the poor performance of these forecasts is that the RMM series contain some amount of day-to-day noise, and the first differencing amplifies this noise relative to the real MJO signal.

In accordance with the recommended forecasting evaluation cycle (see Makridakis *et al*., 1995), a seasonally varying VAR(1) model was then fitted to the original series of full length, that is, 1 January 1979 to 10 November 2003, and checks on the residuals (forecast errors) of this estimated model were made. The checks revealed that the residuals were significantly different from white noise, with some lower lag autocorrelations being significant. This suggests that higher order models may be more appropriate. However, as the higher order VAR models did not satisfy the stationarity conditions, as discussed above, they were not further considered. On the other hand, when a VAR(1) model was fitted to the first differences of the series of full length, and the corresponding nonstationary model was obtained for the original series, the residuals of this model did satisfy the conditions for white noise (i.e. the residuals showed no significant autocorrelations at any lag), but because this model had much poorer forecasting skill than the VAR(1) model applied to the nondifferenced series (Table I), it was also not considered further.

Overall then, as providing skilful, extended-range forecasts is the primary objective, the most satisfactory model is the time-varying VAR(1) model as fitted to the original series. Although this model did not produce residuals that were distinctly white noise, it did satisfy the stationarity condition, and of the models tested, produced the best forecasting statistics in the validation set for forecasts of a useful lead (i.e. beyond day 2).

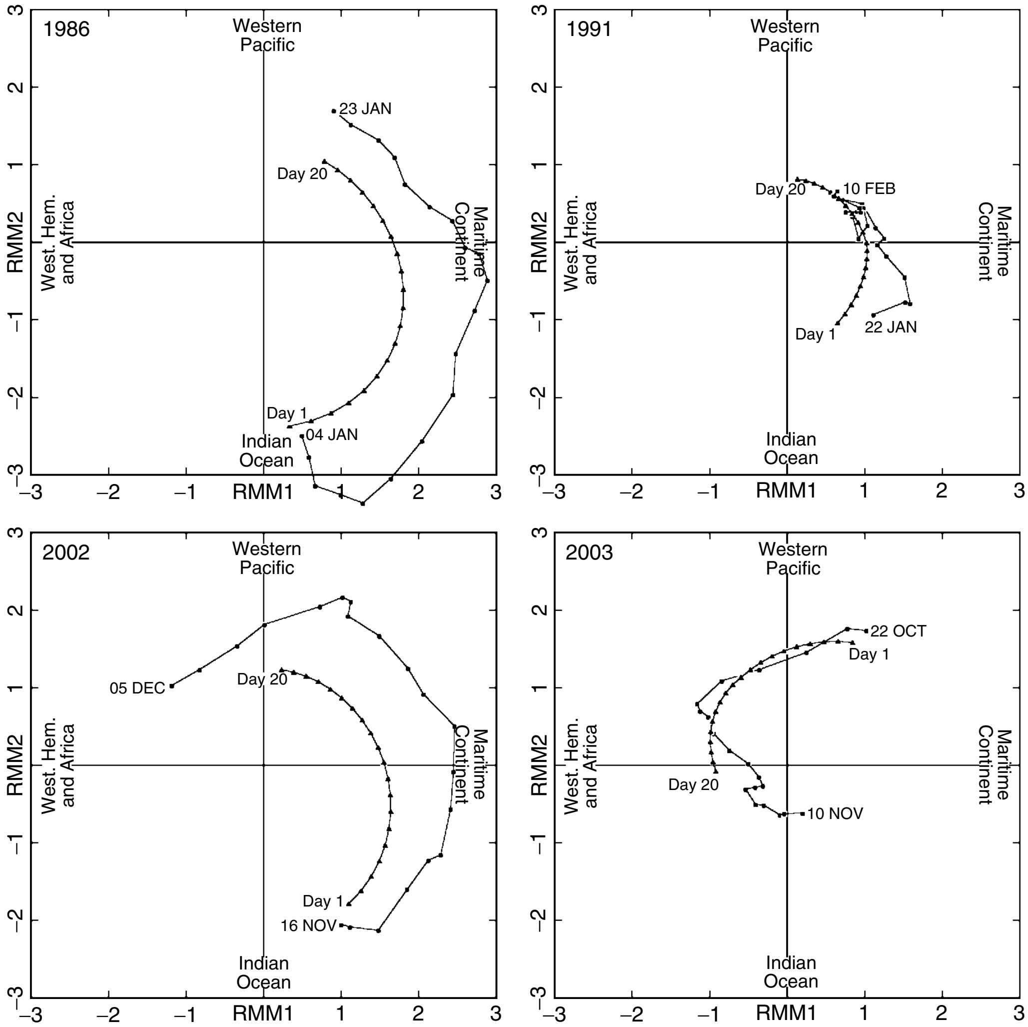

Figure 3. RMM1 and RMM2 forecasts (triangles) and their validating observations (circles) for four example periods, as represented in the two-dimensional phase space they define. Initialisation dates of the 1- to 20-day ahead forecasts are 3 January 1986, 21 January 1991, 15 November 2002, and 21 October 2003. Also labelled are the approximate locations around the earth where the enhanced convective signal of the MJO will be located for that part of the (RMM1, RMM2) phase space (e.g. 'Indian Ocean' for the MJO signal in convection located over the near-equatorial Indian Ocean)

Four example forecasts from the chosen model, each out to 20 days ahead, are displayed in Figure 3. As presented in the phase space defined by RMM1 and RMM2, the forecasted values of the indices, together with their validating observations, are shown. The fourth example provided is for the last 20 days of data available for this study, while the three others were randomly selected. In order, they were initialised on 3 January 1986, 21 January 1991, 15 November 2002 and 21 October 2003. In this phase-space presentation, in all cases both the forecasts and observations trace an approximate anti-clockwise circle about the origin, as is indicative of eastward propagation of the MJO's structure along the equator. In two of the examples (2002 and 2003), the model generates forecasts that propagate more slowly eastward than the observations, while the example from 1986 produces a forecast with much the same amount of propagation as observed, albeit with a weaker amplitude. The example from 1991, on the other hand, has generally weaker amplitudes than the others in both the forecasts and the observations, with the forecast being for greater eastward propagation than observed. None of these differences between the forecasts and validating observations represent systematic problems with the model, however, but are an indication of the large variation of eastward phase speeds, periods and amplitudes, as measured for individual MJO events (e.g. Weickmann and Khalsa, 1990).

5. CONCLUDING REMARKS

In this paper, we have explored methods for forecasting a bi-variate index of the MJO. This involved fitting time-varying VAR models to the original time series and the series of first differences. VAR models of up to order 4 were fitted, and many tests were done to assess the applicability of each of the models. Only the first-order, VAR(1) models passed the strict condition for stationarity, and the applicability of a two-season (summer versus winter) time-varying model was confirmed. Although failing a rigorous test on the residuals, the VAR(1) model on the original (nondifferenced) time series was found to be the most satisfactory for forecasts beyond a few days. For a 15-day forecast, the correlation coefficient between the predicted and validating observations, as averaged for the two individual components of the bi-variate index, was 0.52. This correlation was computed using an independent set of observations that included all seasons.

Such statistical forecasting of an index of the MJO is not new. Other studies that have explored the prediction of an unfiltered daily index of the MJO, as would be available in real time, are those of von Storch and Xu (1990), Lo and Hendon (2000). The former study used the Principal Oscillation Pattern technique to both identify the MJO and make predictions, yielding a correlation skill of 0.4 for 15-day forecasts for all seasons. The latter study, for which a pair of EOFs of OLR was used for identification of the MJO, obtained an average correlation skill of about 0.47 from a lagged linear regression model with the time coefficients of the EOF pair at a single time lag as predictors. When estimating our VAR model using the same period of data as Lo and Hendon (2000), and using the same southern summer validation set as them, we obtain an average correlation skill of 0.48 for a 15-day forecast. Thus a slight advantage over the forecasts of Lo and Hendon (2000) is suggested. Given the similarity in formulation of VAR and linear regression models, however, this advantage is likely a result of our use of an MJO index designed specifically to remove aspects of day-to-day noise (see also WH), and not due to the model. Beating the skill obtained in previous studies, however, has not been the main thrust of this paper.

Instead, the new aspect of this work has been the thorough exploration of an MJO index with the traditional and rigorous methods of time series analysis, in particular, the application of VAR models. Although a VAR model has no strong advantage over lagged regression models in terms of skill, it has the convenience of using a single set of forecast equations to make predictions for multiple forecast horizons.

REFERENCES

Hall JD, Matthews AJ, Karoly DJ. 2001. The modulation of tropical cyclone activity in the Australian region by the Madden-Julian oscillation. *Monthly Weather Review* **129**: 2970–2982.

Lo F, Hendon HH. 2000. Empirical extended-range prediction of the Madden-Julian oscillation. *Monthly Weather Review* **128**: 2528–2543.

Lutkepohl H. 1991. *Introduction to Multiple Time Series Analysis*. Springer-Verlag: Berlin.

Lutkepohl H. 1992. Testing for time varying parameters in vector autoregressive models. In *Readings in Econometric Theory and Practice*, Griffiths WE, Lutekepohl H, Block ME (eds). North Holland: Amsterdam; 243–264.

Makridakis S, Wheelwright C, Hyndman RJ. 1998. *Forecasting: Methods and Applications*, 3rd edn. Wiley: New York.

Madden RA, Julian PR. 1972. Description of global-scale circulation cells in the tropics with a 40-50 day period. *Journal of the Atmospheric Sciences* **29**: 1109–1123.

Schwarz G. 1978. Estimating the dimension of a model. *Annals of Statistics* **6**: 461–464.

von Storch H, Xu J. 1990. Principal oscillation pattern analysis of the tropical 30–60 day oscillation. Part I: definition of an index and its prediction. *Climate Dynamics* **4**: 179–190.

Waliser DE, Lau KM, Stern W, Jones C. 2003. Potential predictability of the Madden-Julian oscillation. *Bulletin of the American Meteorological Society* **84**: 33–50.

Weickmann KM, Khalsa SJS. 1990. The shift of convection from the Indian Ocean to the western Pacific Ocean during a 30-60 day oscillation. *Monthly Weather Review* **118**: 964–978.

Wheeler M, Kiladis GN. 1999. Convectively coupled equatorial waves: analysis of clouds and temperature in the wavenumber-frequency domain. *Journal of the Atmospheric Sciences* **56**: 374–399.

Wheeler MC, Hendon HH. 2004. An all-season real-time multivariate MJO index: development of an index for monitoring and prediction. *Monthly Weather Review* **132**: 1917–1932.

Zhang C, Dong M. 2004. Seasonality in the Madden-Julian oscillation. *Journal of Climate* **17**: 3169–3180.

