Parameterisation of the Atmospheric Boundary Layer: A Retrospective Look

G. D. Hess
Bureau of Meteorology Research Centre, Melbourne, Australia

1. Introduction

The following discussion is a brief sketch of some of the highlights in the development of parameterisation of planetary boundary layers with an emphasis on the atmospheric boundary layer. Space constraints mean that the presentation must be highly selective.

Interest in the atmospheric and oceanic boundary layers was driven by the desire to explain observed phenomena. The oceanographer, Arctic explorer and Nobel Peace Prize winner, Fridtjof Nansen (1902) observed that ice floes drifted 20 – 40^{o} to right of the wind direction in the Northern Hemisphere. He correctly attributed this effect to the physical balance of the Coriolis force, the pressure gradient and friction, but he didn't know how to demonstrate it mathematically, so he consulted his colleague, Vilhelm Bjerknes. Bjerknes handed the problem to his student, Vagn Walfrid Ekman, for his Ph. D. research. Overnight Ekman (1902) obtained his famous spiral solutions for the (steady-state, horizontally homogeneous) drift current and stress profiles. To do this he parameterised the stress as a product of a constant turbulent viscosity multiplied times the vertical shear of the current, in an analogy to molecular case.

$$\{-\overline{uw}, -\overline{vw}\} = K\{\partial U/\partial z, \partial V/\partial z\}. \qquad (1)$$

His boundary conditions specified continuity of stress at the ocean surface and vanishing currents at infinite depth. However when the predictions were compared with observations a problem appeared. The angle between the stress and the current was predicted to be 45^{o}, which was larger than observed.

2. Two-Layer Model

Ekman recognised that K varied with depth. To improve the prediction of the angle, Ekman (1906, 1928) introduced a two-level model, consisting of a thin surface layer in which the current direction was independent of depth and an outer current spiral layer. This allowed the angle to take values from 0^{o} to 45^{o}.

F. Åkerblom (1908) next solved the problem of the wind distribution in the atmospheric boundary layer (apparently independently of Ekman; there is no reference to Ekman's earlier work). His lower boundary condition employed the observed near-surface wind; his upper boundary condition required the wind to approach the geostrophic wind at infinite height. Åkerblom's motivation was to provide a theoretical framework to understand the unique set of wind profiles measurements obtained by Angot (1897) who had instrumented the only fixed atmospheric platform available, the Eiffel Tower in Paris. Åkerblom evaluated K from atmospheric measurements and found it was many hundred thousands of times greater than the molecular value and it had a seasonal variation.

Independently G. I. Taylor (1915) gave a solution to the atmospheric problem by specifying that the wind was parallel to the stress vector near the surface and employing the observed cross-isobaric angle. Taylor's confirmed his theory using the pilot balloon observations taken over the Salisbury Plain by Dobson (1914).

Thus the importance of the surface layer was recognised by Ekman, Åkerblom and Taylor, and had been introduced empirically. We now turn our attention to Ludwig Prandtl (1905) who had introduced the concept of the boundary layer to the engineering and fluid mechanics community. Twenty years later, Prandtl (1925) proposed the concept of the mixing length, over which the momentum of an eddy was conserved, in analogy to the molecular mean path. In the surface layer the eddy diffusivity parameterisation took the form:

$$-\overline{uw} = K\,\partial U/\partial z \sim \overline{\ell w}\,\partial U/\partial z \sim \ell^2(\partial U/\partial z)^2 \sim \ell^2|\partial U/\partial z|(\partial U/\partial z). \qquad (2)$$

In this layer, sometimes called the Prandtl layer, the size of the eddy is proportional to the height above the surface, i.e. $\ell = kz$, where k is von Kármán's constant. Because the stress is nearly constant in the surface layer, this parameterisation led to the famous logarithmic wind profile:

$$kU/u_* = \ell n\,(z/z_0). \qquad (3)$$

Rossby and Montgomery (1935) were the first to develop a two-layer model with $K = kzu_*$ in the (inner) Prandtl layer. In their outer layer K decreased with height,

$$K \sim [1 - (z/h)]^2 \qquad (4)$$

where h is the height of the boundary layer. The Prandtl layer was first coupled with K = constant in the (outer) Ekman-Åkerblom-Taylor layer by Yudin and Shevets (1940). Both models capture the essential physics of the atmospheric boundary layer. The Rossby-Montgomery model is a proto-type for current parameterisation used by the ECMWF (1995) and others, and the Yudin and Shevets model is a proto-type for the University of Washington model (Brown and Liu, 1982).

3. One-layer, Continuous Model

The next stage in sophistication, taken by Al Blackadar (1962) and Heinz Lettau (1962), was to generalise Prandtl's parameterisations for ℓ and K for the boundary layer as a whole:

$$K = \ell^2\{(dU/dz)^2 + (dV/dz)^2\}^{1/2}, \qquad (5)$$

with

$$\ell = kz / [1 + kz / \lambda],\ \lambda = 0.00027\ G/|f|, \qquad (6)$$

(Blackadar), and G the surface geostrophic wind speed, and f is the Coriolis parameter, and

$$\ell = kz / [1 + 33.59\ (|f|z/ku_*)^{5/4}], \qquad (7)$$

(Lettau). Both models approach the logarithmic profile for small z. Blackadar's ℓ approaches a constant value in the upper boundary layer, whereas Lettau's ℓ decreases with height there.

4. Stability effects

Up to this point we have restricted our discussion to neutral conditions. The effects of thermal stratification were quantified by Lewis Richardson (1920) based on energy arguments. He introduced the Richardson number,

$$Ri = (g/\Theta)(\partial\Theta/\partial z)/[(\partial U/\partial z)^2 + (\partial V/\partial z)^2]\ , \qquad (8)$$

Obukhov (1946) employed a semi-empirical analysis to introduce another stability parameter for the surface layer,

$$z/L = -(kgz/\Theta)\ \{(H_0/\rho C_p)\ ) + 0.61E_0/\rho\}/u_*^3, \qquad (9)$$

where L is the length scale of the dynamic turbulence sublayer; Lettau (1949) independently introduced L. Later Monin and Obukhov (1954) derived z/L using dimensional analysis and developed a similarity theory for the surface layer in which scaled mean variables, higher-order moments, and gradients were written in terms of universal functions of stability z/L. Eventually the scaling was extended to the whole boundary layer (see Fig. 1). In the asymptotic limit of free convection ($z/L \rightarrow -\infty$), $K \sim z^{4/3}$, a result found by Prandtl (1932), Obukhov (1946) and Priestley (1954); for great stability ($z/L \rightarrow \infty$), K ~ constant (Obukhov, 1946). It took more than a decade to empirically determine the constants in the Monin-Obukhov theoretical framework.

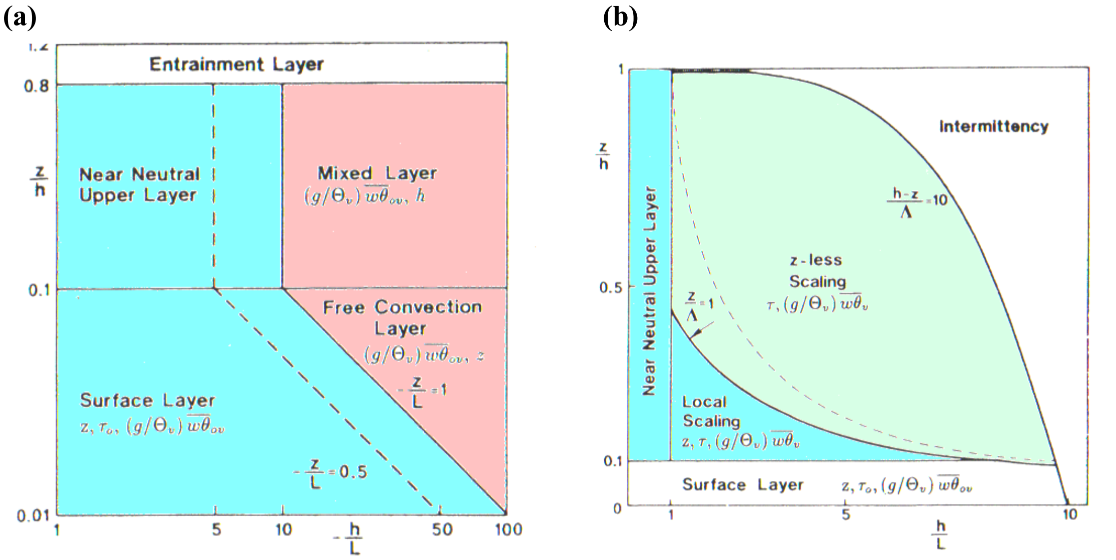

Figure 1. Schematic diagram indicating boundary layer scaling regions and physical parameters of importance. (a) Unstable case (positive buoyancy, L < 0). Instability increases as -h/L increases. The dashed and solid lines indicate the transition region to the convective state. (b) Stable case (negative buoyancy, L > 0). Stability increases as h/L increases. The dashed line is prescribed by z/L = 1. Comparison of the dashed and solid lines shows where the local length scale Λ (based on local fluxes) is approximately equal to the Obukhov length L (based on surface fluxes) (after Holtslag and Nieuwstadt, 1986).

Meanwhile Prandtl and Tollmein (1924) introduced the idea of combining the external variables G, |f| and z_0 into a single non-dimensional number, $Ro = G/|f|z_0$ (later named the surface Rossby number by Lettau) to determine the surface stress. This idea was developed into Rossby number similarity theory by Rossby and Montgomery (1935) and Kazanski and Monin (1960):

$$C_g = u^*/G = k\,\{[A - \ell n\,(u_*/|f|z_0)]^2 + B^2\}^{-1/2} \quad \text{and} \quad \alpha_0 = \arcsin\,\{-\,\mathrm{sign}\;BC_g/k\}, \qquad (10)$$

where sign = +1 in the Northern Hemisphere and −1 in the Southern Hemisphere, and A and B are universal functions of stability and baroclincity. The first determinations of these functions began to appear in the late 1960s.

Kiku Miyakoda of GFDL (see Fig. 2) approached Reg Clarke, based on his field experience, and independently Akira Kasahara of NCAR and Akira Arakawa of UCLA approached Jim Deardorff, based on his large eddy simulations (LES) experience, to develop comprehensive methods to treat the boundary layer.

Figure 2. Kiku Miyakoda's Group at GFDL in 1966. Left to right: Daniel Hembree, Kiku Miyakoda, Reg Clarke and Irv Schulman. Photo taken in front of the National Gallery of Art, Washington, D. C.

Thus by the early 1970s two similar papers appeared recommending two methods for parameterising the boundary layer in large-scale models (Clarke, 1970; Deardorff, 1972) that would take into account the effects of stability. For models with a limited number of vertical levels (i.e. models which were unable to resolve the boundary layer explicitly), Rossby number similarity theory could be used to parameterise the integrated effect of the boundary

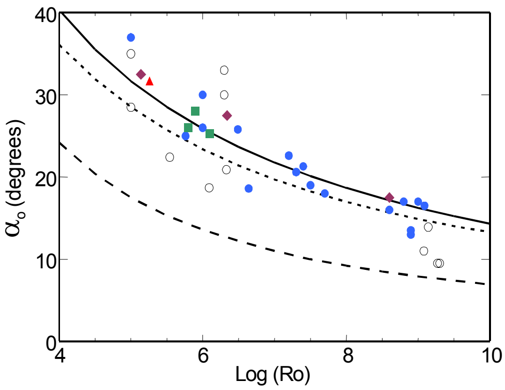

Figure 3. The variation of the cross-isobaric angle, α_0, as a function of the surface Rossby number, Ro, for near-neutral, near-barotropic conditions (dots) in middle to high latitudes (Hess and Garratt, 2002). The triangles indicate data from sites that are non-homogeneous/non-ideal on the microscale; the diamonds refer to climatological data; the squares are experiments with limited neutral data. Other experiments of secondary quality are shown as open circles. The scatter of the data indicated by circles is partly due to sampling and averaging uncertainties, but probably also reflects violations of the assumptions of the theory, e.g. baroclinicity, unsteadiness and advection. The solid line is Eqns (5) and (7) for α_0 (Lettau's model); the dotted line is the result for the simplest two-layer analytical model Eqns (1) – (3); the dashed line is the DNS result (Coleman, 1999).

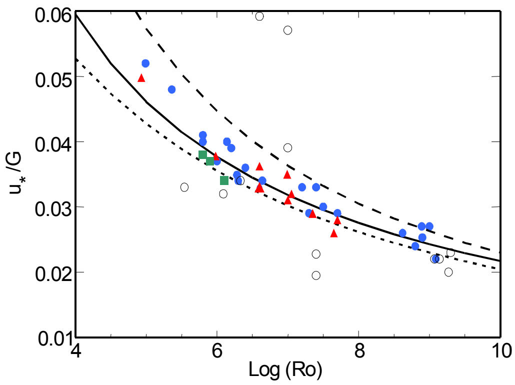

Figure 4. The variation of the geostrophic drag, C_g, as a function of the surface Rossby number, Ro, for near-neutral, near-barotropic conditions (dots) (Hess and Garratt, 2002). The triangles indicate data from sites that are non-homogenous/non-ideal on the microscale; the squares indicate experiments with limited neutral data. Other experiments of secondary quality are shown as circles. The scatter of the data indicated by the circles is partly due to sampling and averaging uncertainties, but probably also reflects violations of the assumptions of the theory, e.g. baroclinicity, unsteadiness and advection. The solid line is Eqns (5) and (7) for C_g (Lettau's model); the dotted line is the result for the simplest two-layer analytical model, Eqns (1) – (3); the dashed line is the DNS result (Coleman, 1999).

layer and to determine the surface fluxes. For models with at least five or six vertical levels the lowest 2 – 3 km, the boundary layer could be explicitly modelled. Louis (1979) devised a scheme to approximate the behaviour of the Monin-Obukhov stability dependence in terms of the bulk Richardson number to improve the speed of the computations.

By the 1990s computer power was sufficient to carry out direct numerical simulations (DNS) of the boundary layer flow in which no subgrid-scale parameterisation was required (e.g. Coleman, 1999). However so far these have been limited to Reynolds numbers (Re) of about 1000, where $Re \equiv GD/\nu$, $D \equiv (2\nu/|f|)^{1/2}$ and ν is the kinematic viscosity. This may be sufficient for Reynolds number similarity to faithfully represent the physics of the energy-containing eddies and the mean velocity, the turbulent kinetic energy and the Reynolds stresses, but further study is required.

Figures 3 and 4 compare the predictions based on various levels of parameterisation: the simple two-layer model of Prandtl-Ekman-Åkerblom-Taylor, the mixing length of Lettau, and the DNS of Coleman. The DNS results are similar to those of other advanced models, such as LES (Mason and Thomson, 1987) where the large eddies of length scale about 100 m are explicitly resolved, and second-order closure models (SOC), such as Wyngaard et al. (1974), where there are predictive equations for the turbulent covariances. These figures indicate that the parameters of the simpler models have been matched to atmospheric observations, whereas the more advanced models base their closure on more general flows or have no free closure parameters. Also the experiments available violate the simplifying assumptions of steady, homogeneous, neutral, barotropic flow required by the advanced models.

The predicted vertical profiles of the stress components for all the models with a low-level inversion over the sea agree with observations (Fig. 5), but over land the simple two-layer model performs better than the advanced TKE model (Fig.6).

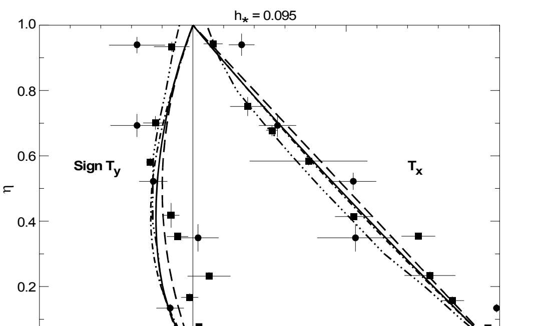

Figure 5. Comparison or predicted and observed profiles of stress components, T_x and T_y over the sea. The squares indicate measurements from the KONTUR Experiment (Grant, 1992) and the circles from the Marine Stratocumulus Experiment (Brost et al, 1982). The solid lines are the predictions from the TKE model (Krishna and Arya, 1981), the dashed-dotted lines from Prandtl-Ekman-Åkerblom-Taylor model, the dashed lines from the SOC model (Brost and Wyngaard, 1978) and the dash-triple dotted lines from the LES model of Moeng and Sullivan (1994). $h_* = h|f|/u_*$. (After Hess, 2004).

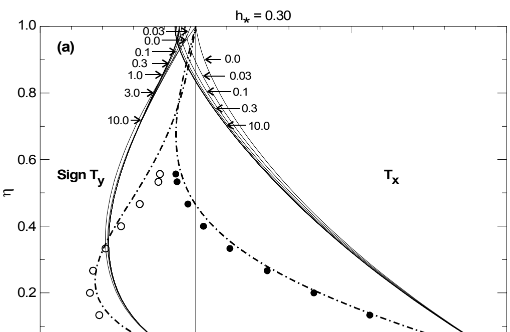

Figure 6. Comparison of the Prandtl-Ekman-Åkerblom-Taylor model (dash-dotted lines) and the TKE model (solid lines) predictions for the components of geostrophic departure over land. Observations from the Leipzig experiment are shown in circles. The effect of different rates of entrainment W_{e*} from 0.0 – 10.0 are indicated (After Hess, 2004).

Further work is required to understand the differences between the predictions of advanced models and observations. The complexity of the atmosphere and its variability makes comparisons difficult to interpret. However, some progress towards overcoming these difficulties is found in a recent (as yet unpublished) paper, which provides improvements in both similarity theory and LES modelling (Zilitinkevich and Esau, 2004, personal communication).

References

Åkerblom, F. 1908. Recherches sur les courants les plus bas de l'atmosphère au-dessus de Paris. *Nova Acta Reg. Soc. Sc., Upsala,. Ser. IV,* **2**, 1 – 45.

Angot, A. 1897. Résumé des observations anémométriques faites au Bureau central et à la Tour Eiffel pendant les six années 1890-95. *Annales du Bureau centr. Météorol. de France*, **1**, B. 171.

Blackadar, A. K. 1962. The vertical distribution of wind and turbulent exchange in a neutral atmosphere, *J. Geophys. Res.* **67**, 3095-3102.

Brost, R. A. and Wyngaard, J. C. 1978. A model study of the stably stratified planetary boundary layer. *J. Atmos. Sci*., **35**, 1427-1440.

Brost, R. A., Wyngaard, J. C. and Lenschow, D. H. 1982. Marine stratocumulus layers. Part II: Turbulence budgets. *J. Atmos. Sci*., **39**, 818-836.

Brown, R. A. and Liu, T. 1982. An operational large-scale marine planetary boundary layer model. *J. Appl. Meteorol*., **21**, 261-269.

Clarke, R. H. 1970. Recommended methods for the treatment of the boundary layer in numerica models. *Aust. Met. Mag*., **18**, 51-71.

Coleman, G. N. 1999. Similarity statistics form a direct numerical simulation of the neutrally stratified planetary boundary layer. *J. Atmos. Sci*., **56**, 891-900.

Deardorff, J. W. 1972. Parameterization of the planetary boundary layer for use in general circulation models*. Mon. Wea.. Rev*., **100**, 93-106.

Dobson, G. M. B. 1914. Pilot balloon ascents at the Central Flying School, Upavon, during the year 1913. *Q. Jl. R. Met. Soc*., **40**, 123-135.

ECMWF Research Department. 1995. *ECMWF Forecast Model: Physical Parametrization*, Fourth Edition, ECMWF, Shinfield Park, UK.

Ekman, V. W. 1902. Om jordrotationens inverkan pä vindströmmar i hafvet. *Nyt Mag. f. Naturvidenskab*, **40**.

Ekman, V. W. 1906. Beiträge zur Theorie der Meeresströmungen. II. Die Strömungen, die von dem Winde under der Erdrotation allein erregt werden. *Annln. Hydrogr*., **34**, 472-484.

Ekman, V. W. 1928. Eddy-viscosity and skin-friction in the dynamics of winds and ocean-currents. *Mem. R. met. Soc*., **2**, 161-172.

Hess, G. D. 2004. The neutral, barotropic planetary boundary layer, capped by a low-level inversion. *Bound. Lay. Met*., **110**, 319-356.

Hess, G. D. and Garratt, J. R. 2002. Evaluating models of the neutral, barotropic planetary boundary layer using integral measures: Part 1. Overview. *Bound. Lay. Met*., **104**, 333-358.

Holtslag, A. A. M. and Niewstadt, F. T. M. 1986. Scaling the atmospheric boundary layer. *Bound. Lay. Met*., **36**, 201-209.

Kasanski, A. B. and Monin, A. S. 1960. A turbulent regime above the ground atmospheric layer. *Izv. Geophys. Ser*. **1**, 110-112.

Khrisna, K. and Arya, S. P. S. 1981. Wind structure in a neutral, entraining PBL capped by a low-level inversion. *Prep. Fifth Symp. Turbulence, Diffusion Air Pollution*, 9 – 13 March 1981, Atlanta, GA, American Meteorological Society, Boston, 65-66.

Lettau, H. H. 1949. Isotropic and non-isotropic turbulence in the atmospheric surface layer. *Geophys. Res. Paper No. 1*, Air Force Cambridge Research Laboratory, Cambridge, MA, 13-84.

Lettau, H. H. 1962. Theoretical Wind Spirals in the Boundary Layer of a Barotropic Atmosphere, *Beiträge Phys. Atmos*., **35**, 195-212.

Mason, P. J. and Thomson, D. J. 1987. Large eddy simulation of the neutral-static-stability planetary boundary layer. *Q. J. R. Met. Soc*., **113**, 413-443.

Moeng, C.H. and Sullivan, P. P. 1994. A comparison of shear- and buoyancy-driven planetary boundary layer flows. *J. Atmos. Sci.*, **45**, 3575-3587.

Monin, A. S. and Obukhov, A. M. 1954. Basic turbulent mixing laws in the atmospheric surface layer. *Trudy Geofiz. Inst., AN SSSR*, No. 24 (151), 163-187.

Nansen, F. 1902. Oceanography of the North Polar Basin. *The Norwegian North Polar Expedition 1893-96*, **3**, Kristiania.

Obukhov, A. M. 1946. Turbulence in thermally inhomogeneous atmosphere. *Trudy In-ta Teoret. Geofiz. AN SSSR*, **1**, 95-115.

Prandtl, L. 1905. Über Flüssigkeits-bewegung bei sehr kleiner Reibung. *Proc. 3rd Internat. Math. Congr*., Heidelberg 1904.

Prandtl, L. 1925. Bericht über Unter-suchungen zur ausgebildeten Turbulenz. *Z. angew. Math. Mech*., **5**, 136-139.

Prandtl, L. 1932. Meteorologische Anwendungen der Strömungslehre, *Beitr. Phys. Frei. Atmos*., **18**, 188-202.

Prandtl, L. and Tollmien, W. 1924. Die Windverteilung uber dem Erdboden, errechnet aus den Gesetzen der Rohrstromung, *Z. Geophys*., **1**, 47-55.

Priestley, C. H. B. 1954. Convection from a large horizontal surface, *Aust. J. Phys*., **6**, 279-290.

Richardson, L. F. 1920. The supply of energy from and to atmospheric eddies. *Proc. R. Soc. London, Series A*, **XCVII**, 354-373.

Rossby, C. G. and Montgomery, R. B. 1935. The Layers of Frictional Influence in Wind and Ocean Currents. *Papers Phys. Oceanogr. Meteorol*., **3**, No. 3, 101 pp.

Taylor, G. I. 1915. Eddy motion in the atmosphere. *Phil. Trans. R. Soc. London, Series A*, **CCXV**, 1-26.

Wyngaard, J. C., Coté, O. R. and Rao, K. S. 1974. Modeling the atmospheric boundary layer. *Adv. Geophys*., **18A**, 193-211.

Yudin, M. I. and Shevets, M. Ye. 1940. Stationary model of the height distribution of wind in a turbulent atmosphere. *Trans. (Trudy) Chief Geophys. Observ. (GGO),* No. 31, 42-52.