ASSESSING ERRORS IN URBAN DRAINAGE SYSTEM MODEL OUTPUT FLOWS DRIVEN BY RADAR AND OTHER RAINFALL DATA

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Abstract

An hydraulic model of an urban drainage system with a Bayesian post processor is used to assess the uncertainty in model output flows associated with the use of measurements of rainfall made using raingauges, microwave link attenuation measurements and weather radar.

Key Words: Hydraulic model, microwave link, raingauges, radar, Bayesian statistics

1.Introduction

Water Companies throughout the UK are under constant pressure to reduce the frequency of combined sewer overflows (CSOs) to natural water courses from urban drainage systems (UDS). The management of storm-water through the UDS may be based upon the use of mathematical models of the hydraulic system. However, the realism of the performance of these models depends to a large measure upon the accuracy of their rainfall input. The observing systems used to measure the rainfall distribution include rain gauges, weather radar and, possibly, line integrated measurements inferred from the attenuation produced by rainfall along microwave communication links operating at one and two different frequencies. In this paper we discuss a method of assessing the errors in UDS model output flows arising from the use of these different observing systems as a function of rainfall event type.

The analysis has been carried out for the UDS serving the town centre of Bolton, Greater Manchester in North West England. Bolton receives a total annual rainfall of about 1300mm, predominantly from systems moving from the Western Atlantic ocean. The UDS area is some 93km^2 serving a population of 260,000. Rainfall runoff is transported together with domestic, commercial and industrial effluent along a network of approximately 1200km of pipes having diameters from 150mm to 1,500mm to an outlet treatment works at Ringley. Built mostly between 1890 and 1930, the UDS often has insufficient capacity to deal with this combined flow during only moderately heavy rainfall events resulting in many occurrences of flooding.

The town centre UDS is one of the largest sub catchments draining an area of approximately 21km^2, and serving a population of 62,000. Flooding problems in this area are amplified by the steep nature of the system there being a 60m drop over a horizontal distance of 8,500m. Three large off-line tanks have been built to reduce CSOs by receiving flows diverted from the main sewer. These tanks have capacities of 2,000m^3 (Ladybridge) and 10,000m^3 (Spa Road and Water Street). A network of 22 tipping bucket rain gauges has been installed in this area, which is also within the quantitative measurement range of the C-band weather radar located at Hameldon Hill some 24km from the sub catchment (Fig 1). A number of microwave links for measuring rainfall have also been installed in this area (Fig 1). We do not discuss the microwave link technique here,

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but details are to be found in Hardaker et al (1977) and Holt et al (2000).

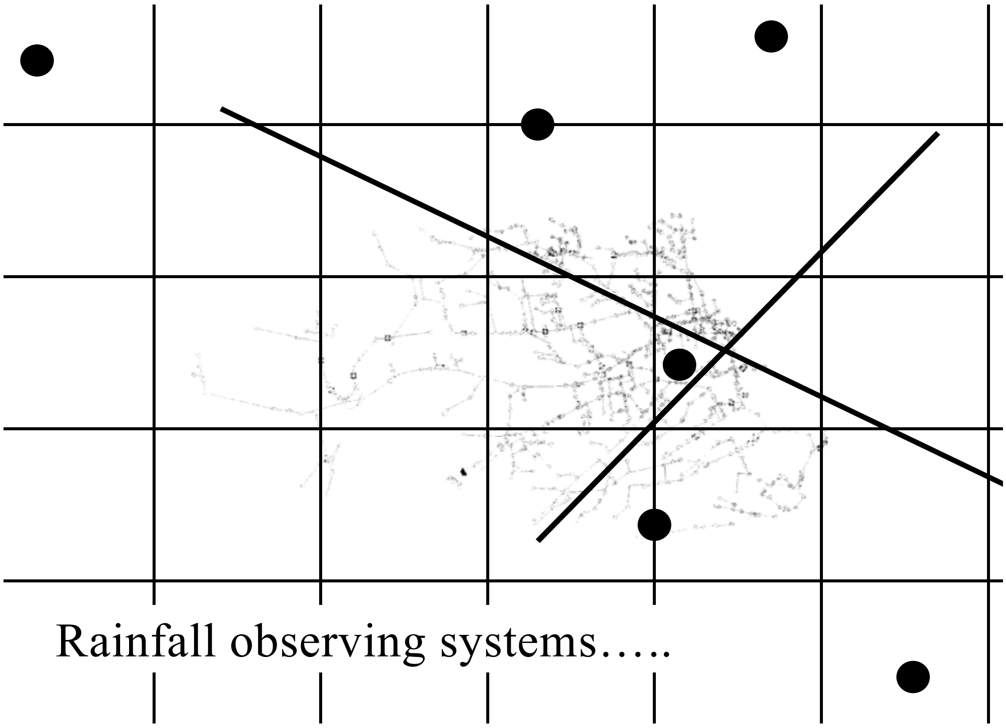

Figure 1: The Bolton town centre UDS each dot representing a model node. Also shown are raingauge locations (large dots), two microwave links and the 2km x 2km radar grid boxes.

2. The hydraulic model

The UDS modelling is performed using the propriety system Hydroworks™ . This is the latest version of a commercial package based upon the WALLRUS system developed by Wallingford Software. The drainage system is defined in terms of its location, shape, dimensions, surface roughness, head loss and depth-flow relationships. The computational nodes are located (manholes, basins and outlets) before being linked by conduits (pipes, open channels and culverts). Throughout the system other ancillary features exist (overflow weirs, pumps, non-return valves, flow restrictions, off-line storage tanks and other controllable structures). The Bolton town centre model comprises 1,289 nodes, 1,315 conduits, 45 weir flumes and orifices, 32 overflows, 50 online storage tanks, 3 offline storage tanks and 15,234 additional computational nodes.

3. Assessing the performance of the hydraulic model

An objective function may be used to assess the ability of the hydraulic model to produce an acceptable hydrograph using a particular rainfall observing system, compared with that produced using some other observing system. Several such functions may be used. In the present work we adopted the function defined by Yuan et al (1999) as follows,

$$F = e^{-\varepsilon} \qquad (1)$$

where $$\varepsilon = \frac{1}{\delta + 1} \sum_{i=t-\delta}^{t} \left| \frac{\Omega_I - Q_I}{\Omega_I} \right|$$

Ω_I = modelled flow using the full raingauge network at time step i

Q_I = modelled flow using another network at time step i

t = time $\qquad$ δ = event duration having i time steps

F has a maximum value of 1.0 for perfect agreement between the hydrographs. Note that the performance of specific observing systems are compared here with the performance achieved using the full rain gauge network. This is necessary as continuous flow measurements are not made at the output of the UDS. However we discuss later a method of assessing performance taking account of the ability of the model to represent actual flows.

We may compare values of F for different rainfall event types defined in terms of an objective parameter VR representing rainfall variabilityas follows,

$$VR = \Sigma^{G}_{j=1} \left(1/ N_j\text{-}1 \Sigma^{N}_{i=1} (I_{ij} - \overline{I_j})^2 \right)^{1/2} / \ \overline{I_j} \ ) / G \qquad (2)$$

where $I_{I,j}$ = rainfall intensity measured by gauge j at time step i

$\overline{I_j}$ = mean rainfall measured by gauge j

N_j = number of time steps between the start and end of rainfall event measured by gauge j

G = number of gauges

The larger the value of VR, the more convective is the event. However, an alternative methodology is to use the Vertical Profile of Reflectivity (VPR) evaluated within 75km (say) of the radar site to define the rainfall event type. The VPR for stratiform events exhibit a distinct bright band enhancement and a characteristic fall off in the snow aloft. Generally in convective rainfall the VPR remains more constant with height.

As mentioned previously unfortunately flows are not routinely measured at the UDS output, although from time to time measurement campaigns have been carried out enabling an assessment of the ability of the model to predict output flows to be assessed. Fig 2 shows a comparison of the model output flows with actual measurements made May-June 1999. On average the model is capable of reproducing actual flow albeit with moderate sized uncertainty.

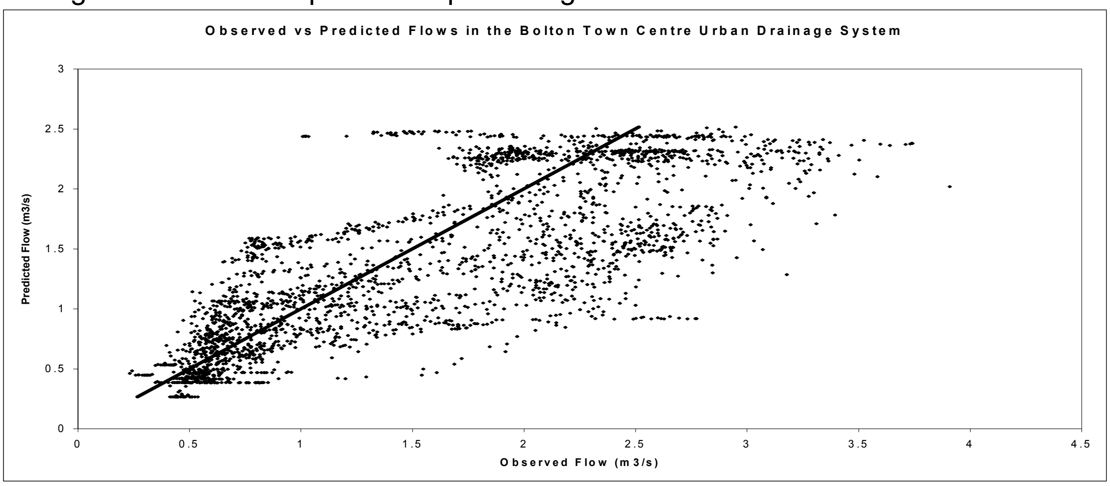

Figure 2: Performance of Hydroworks^{TM} for the Bolton town centre UDS May-June 1999

Whilst the evaluation of F provides information on average model performance for a specific rainfall event variability it does not define the total uncertainty including errors in the model itself associated with that performance. One approach to estimating the total uncertainty of the hydraulic model flows is to couple the model to a Bayesian post processor as described by Krzysztofowicz (1999a).

Following the tutorial text of Krzysztofowicz (1999b), let W denote the predictand, and x denote a deterministic forecast of the predictand produced by the deterministic hydraulic model. Uncertainty remains about the realisation of W given its forecast x. To quantify the uncertainty in the forecast a Bayesian processor may be used with the forecast x as input. This processor quantifies the uncertainty in terms of a probability density function of W. The forecast x is treated as a realisation of variate X , and principles of Bayesian inference are applied to (X, W).

The uncertainty about the predictand that exists before the preparation of a forecast is quantified in terms of a prior density g of W. The prior uncertainty is associated with that arising from the nature of the observing system providing the rainfall input to the deterministic model. The predictive capability of the deterministic model (model errors) is characterised in terms of a family of conditional densities $\{f(.|w)$: all $w\}$ where $f(.|w)$ is the density of forecast variate X, conditional on the hypothesis that the realisation of the predictand is W = w.

The Bayesian processor outputs the following:

(a)The variability of the forecast variate X in terms of the predictive density K specified by the total probability law,

$$K(x) = \int f(x|w)\, g(w)\, dw \qquad (3)$$

(b) The uncertainty about the predictand W, conditional upon forecast X = x in terms of the posterior $\eta\,(.|x)$ given by Bayes theorem,

$$\eta\,(w|x) = f(x|w)\, g(w) / K(x) \qquad (4)$$

Hence, the family $\{\eta\,(.|x)$: all $x\}$ of the posterior densities provides an assessment of uncertainty about predictand W for every possible forecast x.

If we assume that all densities are parametric, the Bayesian processor is referred to as a normal-linear Bayesian processor in which the prior density g is normal with mean and variance,

$$E\,(W) = M \qquad (5)$$

$$Var\,(W) = S^2 \qquad (6)$$

And the conditional density $f\,(.|w)$ is normal with mean and variance,

$$E\,(X|W = w) = aw + b \qquad (7)$$

$$Var\,(X|W = w) = \sigma^2 \qquad (8)$$

Therefore it follows that the predictive density K is normal with moments,

$$E\,(X) = aM + b \qquad (9)$$

$$Var\,(X) = a^2 S^2 + \sigma^2 \qquad (10)$$

And the posterior density $\eta(.|x)$ is normal with moments,

$$E(W \mid X = x) = Ax + B \tag{11}$$

$$Var(W \mid X = x) = T^2 \tag{12}$$

Where $A = aS^2 / (a^2S^2 + \sigma^2)$ $\quad B = (M\sigma^2 - abS^2) / (a^2S^2 + \sigma^2)$ (13)

$$T^2 = \sigma^2 S^2 / (a^2S^2 + \sigma^2)$$

Hence, the posterior parameters (A,B,T^2) can be obtained directly from the prior parameters(M, S^2) and the likelihood parameters (a,b,σ^2).

For perfect forecasts $a = 1$, $b = 0$ and $\sigma^2 = 0$, and for forecasts which are randomly generated from an arbitrary normal distribution with mean N and variance R^2 then $a = 0$, $b = N$ and $\sigma^2 = R^2$. The slope a measures output information i.e. the "signal" carried by the output, and the conditional variance σ^2 measures output uncertainty i.e. the "noise" in the output. An assessment of the ability of the Hydroworks™ model to represent output flow from the UDS was discussed earlier (Fig 2). A similar analysis for a snowmelt runoff model is outlined in Krzysztofowicz (1999b). On average Hydroworks™ does produce acceptable flows remembering that there are likely to be significant errors in the flow measurements. Also the F factor for the data in Fig 2 (not shown) are approximately constant with rainfall variability. Hence, we will assume $a = 1$ and $b = 0$. Fig 2 shows that σ is certainly not zero so representing the significant uncertainty in the model representations of flow. Therefore taking these values of a and b we may modify equations (11) and (12),

$$\begin{aligned} E(W \mid X = x) &= Ax + B \\ &= \frac{aS^2(x - b) + M\sigma^2}{a^2S^2 + \sigma^2} \\ &= \frac{S^2x + M\sigma^2}{S^2 + \sigma^2} \end{aligned} \tag{14}$$

$$\begin{aligned} Var(W \mid X = x) &= T^2 \\ &= \frac{\sigma^2 S^2}{a^2S^2 + \sigma^2} \\ &= \frac{\sigma^2 S^2}{S^2 + \sigma^2} \end{aligned} \tag{15}$$

Equations (14) and (15) are equivalent to the expressions given by Lorenc and Hammon (1988).

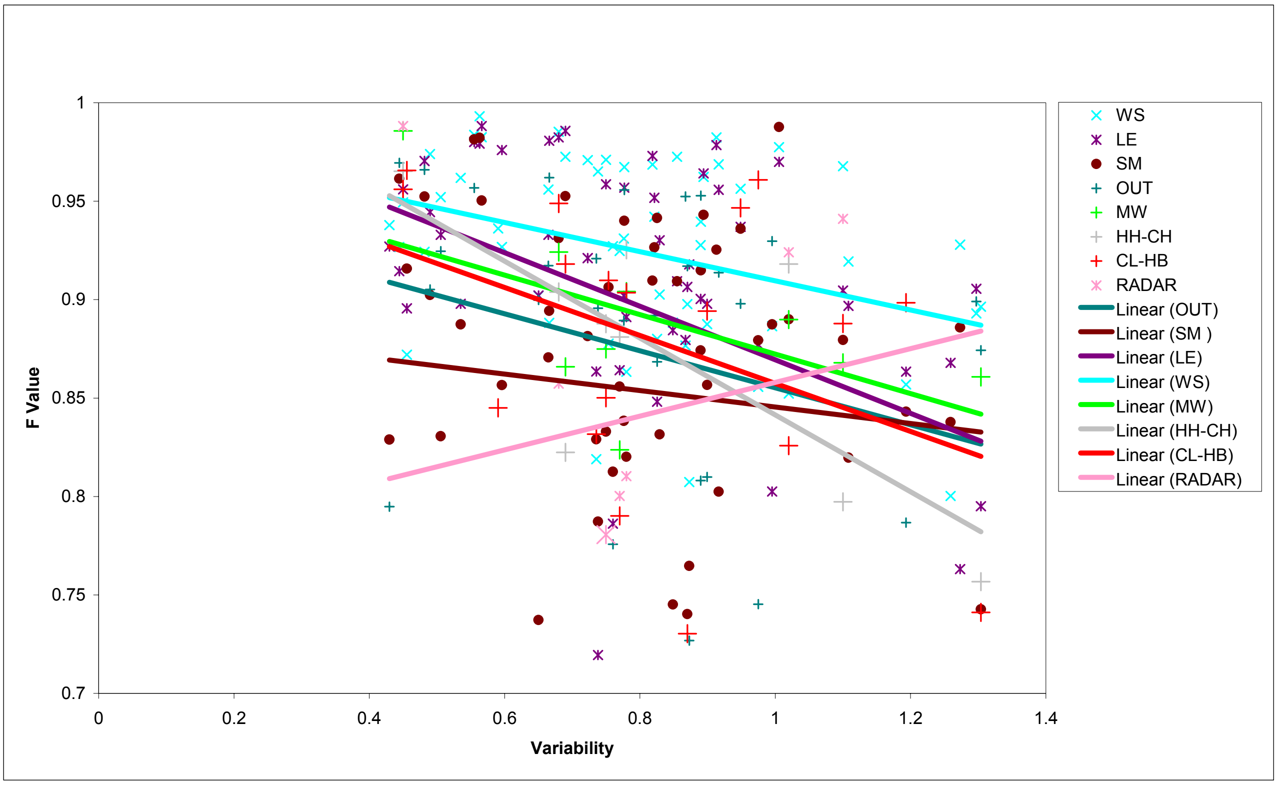

Figure 3: Variation of F with rainfall variability, VR, for observing systems using rain gauges (SM, LE, WS (see Fig 1), OUT – rain gauges outside the catchment (see Fig 1)), microwave links (MW) (CL – Clarkes Hill; HB – Height Barn; HH – Heather Hall (see Fig 1)) and the Hameldon Hill weather radar (RADAR).

4. Analysis of uncertainty

The observing system error in F, x and its variance σ^2 for each rainfall event variability may be evaluated from graphs such as those shown in Fig 3, which has been constructed for rain gauge networks, microwave links and radar model input.

Evaluation of equations (14) and (15) in rainfall variability bands converting the results from F factors to percentage errors in the model output flows are shown in Fig 4. It is seen that using rain gauges within the catchment produces flows with the smallest errors when the rainfall variability is low (stratiform), but the errors get larger as the rainfall variability increases. The microwave links outperform the use of rain gauges from outside the catchment, but not as well as gauges located within the catchment. Radar performance improves as the rainfall variability increases.

5. Concluding remarks

The Bolton town centre catchment is very small, and the data sets used in the analysis are not large, particularly the radar data set. Hence, the results shown in Fig 4 are likely to be pessimistic. For a large river catchment it is expected that the evaluation analysis based upon a Bayesian post processor will reveal enhanced performance of an hydrological model using radar data. This is the subject of on-going research. However, the use of a Bayesian post processor offers the prospect of providing real-time uncertainty forecasts associated with model output flows.

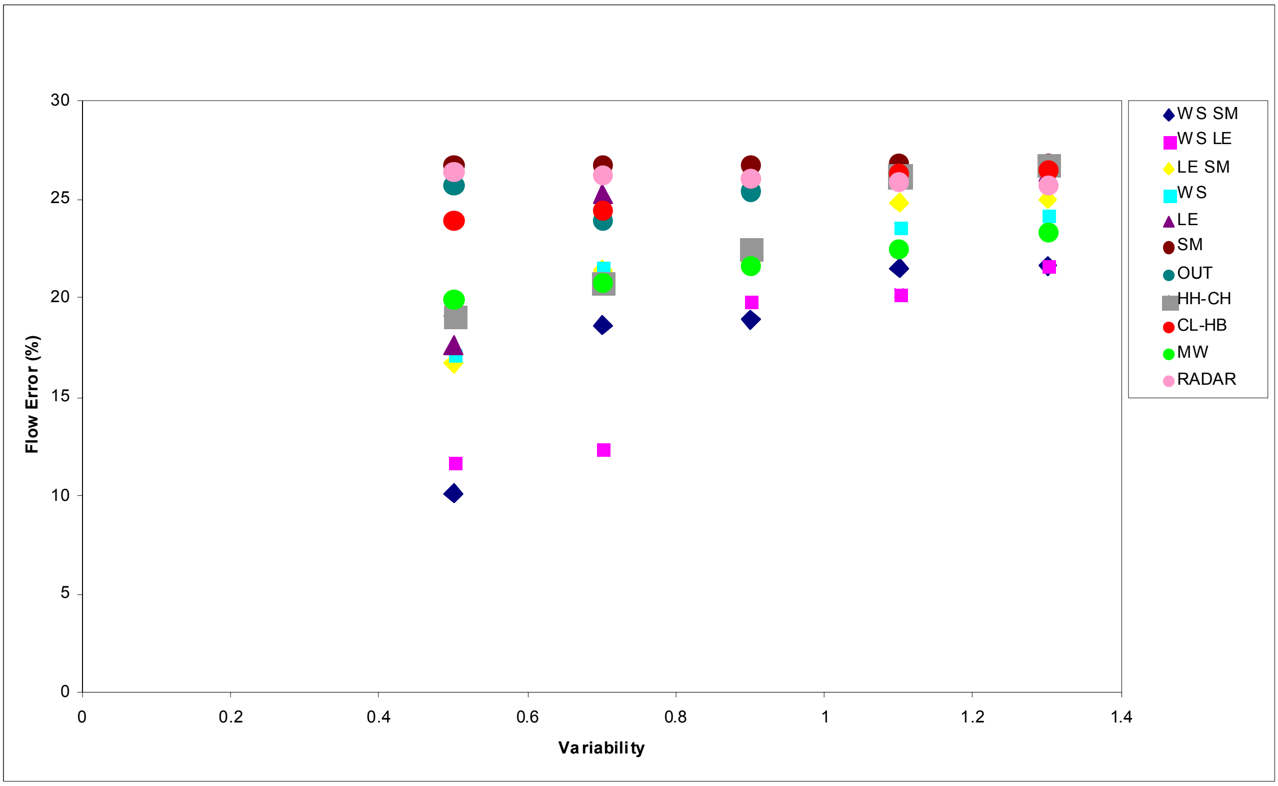

Figure 4: Percentage error in model output flows for the Bolton town centre UDS as a function of rainfall variability for the different observing systems noted in the legend to Fig 3.

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